U.G. 1st Semester Examination - 2018

PHYSICS

(HONOURS)

Course Code : PHSH/CC-T-I

Mathematical Physics-1

Full Marks : 40   Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **five** questions : $2\times5=10$

   i) Write down the properties of delta function.

   ii) Using vector method prove that the angle in a semi circle is a right angle.

   iii) Find the torque about the point O(3, –1, 3) of a force $\vec{F}(4,2,1)$ passing through the point P(5,2,4).

   iv) Prove that

   $$\left[(\vec{B}\times\vec{C}).\{(\vec{C}\times\vec{A})\times(\vec{A}\times\vec{B})\}\right]=\left[\vec{A}.(\vec{B}\times\vec{C})\right]^2$$

   v) Show that $r^n\vec{r}$ is solenoidal only when $n=-3\,(r\neq0)$.

[Turn over]

vi) State Stoke's theorem.

vii) Applying Gauss divergence theorm, evaluate $\iint_S \frac{\vec{r}.\hat{n}}{r^3} ds$, where S represents any closed surface enclosing volume V.

viii) Prove that a cylindrical coordinate system is orthogonal.

Answer any **two** questions : 5×2=10

2. i) Evaluate $\lim_{x \to 0} \frac{\log(1+\alpha x)}{e^{2x}-1}$.

ii) Examine the continuity of the function

$$f(x) = \begin{cases} \frac{|\sin x|}{x} & \text{when } x \neq 0 \\ 1 & \text{when } x = 0 \end{cases}$$

at x= 0. 2+3

3. i) Explain the geometrical interpretation of $\vec{\nabla}\phi$, where $\phi(x, y, z)$ is a scalar function.

ii) The equation to the trajectory of a particle is given by $\vec{r} = \hat{i}e^t \cos t + \hat{j}e^t \sin t$. Find out an expression for the acceleration of the particle and show that it is at right angle to $\vec{r}$. 2+3

4. i) If $\vec{A} = (y - 2x)\hat{i} + (3x + 2y)\hat{j}$, compute the circulation of $\vec{A}$ about a circle C in the xy-plane with centre at the origin and radius 2, C is traversed in the positive direction.

ii) Prove that $\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2 \vec{A}$. 2+3

5. i) Calculate arc length and volume element in orthogonal curvilinear co-ordinate space.

ii) Prove that differential operator $\vec{\nabla} = \frac{\hat{e}_1}{h_1}\frac{\partial}{\partial u_1} + \frac{\hat{e}_2}{h_2}\frac{\partial}{\partial u_2} + \frac{\hat{e}_3}{h_3}\frac{\partial}{\partial u_3}$, in orthogonal curvilinear co-ordinate space. (1+1)+3

Answer any **two** questions : 10×2=20

6. i) Solve the differential equation $y^3 \frac{dy}{dx} + \frac{y^4}{x} = x$.

ii) Solve the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 13y = 0$$

with initial boundary conditions y(0)=2 and

$$\frac{dy(0)}{dx} = 1.$$

iii) Solve the boundary value problem $\frac{\partial u(x,y)}{\partial x} = 4\frac{\partial u(x,y)}{\partial y}$,

given $u(0, y)=8e^{-3y}$. by the method of separation of variables. 3+4+3

7. i) For the function $\phi(x,y)=\frac{x}{x^2+y^2}$, find the magnitude of the directional derivative along a line making an angle 30° with the positive x-axis at (0,2).

ii) Show that the vector field $\vec{V}=\frac{-x\hat{i}-y\hat{i}}{\sqrt{x^2+y^2}}$ is a sink field.

iii) Find the constants a, b, c, so that $\vec{\nabla}\times\vec{v}=0$, where $\vec{v}=(x+2y+az)\hat{i}+(bx-3y-z)\hat{j}+(4x+cy+2x)\hat{k}$. Show that $\vec{v}$ can be expressed as the gradient of a scalar function and find the scalar function.

3+3+4

8. i) If $\vec{A}=-\hat{i}y+\hat{j}x$ and C is a closed curve in the xy-plane, then applying Stoke's theorem prove that $\oint \vec{A}.d\vec{r}=2S$, where S is the area enclosed in the xy-plane by C. Hence, prove that area of an ellipse in πab.

ii) Evaluate the integral $\oint_C \left[\left(xy - x^2\right)dx + x^2 y\, dy\right]$.

over the triangle bounded by the lines y=0, x=1 and y = x and verify Green's theorem.

iii) If S is any closed surface enclosing a volume V and $\vec{A} = ax\hat{i} + by\hat{j} + cz\hat{k}$, prove that

$\iint_S \vec{A}.\hat{n}ds = (a+b+c)v$. (2+2)+(2+2)+2

9. i) Prove that $\hat{e}_1 = h_2 h_3 \left(\vec{\nabla}u_2 \times \vec{\nabla}u_3\right)$.

ii) Obtain expression for $\vec{\nabla}.\vec{A}$ and $\nabla^2\phi(u_1, u_2, u_3)$ in orthogonal curvilinear co-ordinate system.

iii) Define the Dirac delta function $\delta(x)$. Find the value of $\int_{-\infty}^{\infty} \delta(x) e^{ikx} dx$. $1\frac{1}{2}$+(3+3)+$2\frac{1}{2}$
